**Unit-II**

**Discrete Time System Analysis**

**Session-1 Date: 27.07.13 , 2nd hour, Time: 10.05 am-10.55 am**

**Recap: Mathematical representation of signals**

**Suggested Activity: Rapid fire**

1. **Various Representation of Signals:** Graphical, Sequence, Tabular & Functional
2. **Condition for Unit impulse response:** δ(n) = 1 for n = 0, δ(n) = 0 for n ≠ 0
3. **Condition for Unit step sequence:** u(n) = 1 for n ≥ 0, u(n) = 0 for n < 0
4. **Elementary Discrete-time signals:** Unit step, ramp, impulse sequence.
5. **Condition for Energy & Power signal:** E=finite & P=0, E=infinite & P=finite.

**Content: Introduction to Z-transform**

**Suggested Activity: Board Activity**

* The z-transform of a discrete-time signal x(n)

 X(z) = $\sum\_{n=-\infty }^{\infty }x(n)z^{-n}$

* The z-transform of a signal x(n) is

 X(z) = Z { x(n) }

* The relationship between x(n) and X(z) is

 z

 x(n) ↔ X(z)

**Conclusion: Introduction to Z-transform**

**Suggested Activity:** Pick& Answer

There are variety of questions based on the content of the session and any one of the learner is asked to pick the letter and the corresponding question to be answered**.**

1. LTI System
2. Z-transform
3. Region of Convergence
4. Inverse Z-transform
5. If the learner choose **letter d** then the question is **Inverse Z-transform** and the answer is **X(z) = Z { x(n) }**

**Ref :** <http://en.wikibooks.org/wiki/Digital_Signal_Processing/Z_Transform>

**Session-2 Date: 30.07.13 , 5th hour, Time: 01.30 pm-02.20 pm**

**Recap: Introduction to Z-transform**

**Suggested Activity: Quiz**

1. z-transform is **X(z) =** $\sum\_{n=-\infty }^{\infty }x(n)z^{-n}$
2. Region of Convergence means **X(z) attains a finite value.**
3. Zk becomes unbounded for **Z = ∞**
4. Z-k becomes unbounded for **Z = 0**

**Content: Z-transform and its properties**

**Suggested Activity: Match the following**

 **Column-A Column-B**

1. **Linearity d X(z) / dz 5**
2. **Time shifting X(a-1z) 3**
3. **Scaling a1X1(z)+a2X2(z) 1**
4. **Time reversal z-kX(z) 2**
5. **Differentiation X(z-1) 4**

**Ref :** <http://inst.eecs.berkeley.edu/~ee123/fa12/Notes/Lecture04.pdf-properties>

**Conclusion: Z-transform and its properties**

**Suggested Activity: Brainstorming**

1. **Linearity :** x(n) = a1x1(n)+a2x2(n) ↔ X(z) = a1X1(z) + a2X2(z)
2. **Time shifting :** x(n-k) ↔ z-k X(z)
3. **Scaling :** anx(n) ↔ X(a-1z)
4. **Time reversal :** x(-n) ↔ X(z-1)
5. **Differentiation :** nx(n) ↔ -z dX(z) **/** dz

**Ref :** <http://math.fullerton.edu/mathews/c2003/ZTransformIntroMod.html>

**Session-3 Date: 31.07.13 , 3rd hour, Time: 11.10 am-12.00 pm**

**Recap: Z-transform and its properties**

**Suggested Activity: Questions & Answers**

1. **State the convolution of two sequences.**

x(n) = x1(n)\* x2(n) ↔ X(z) = X1(z)X2(z)

1. **Define multiplication of two sequences.**

x(n) = x1(n)x2(n)↔ X(Z) = 1 / 2π ∫c X1(v) X2(z/v) v-1 dv

1. **Define the property of linearity.**

x(n) = a1x1(n)+a2x2(n) ↔ X(z) = a1X1(z)+a2X2(z)

1. **Define the property of time shifting.**

x(n-k) ↔ z-k X(z)

1. **Define the property of Scaling in z-domain.**

anx(n) ↔ X(a-1z)

**Content: Inverse z-transform**

**Suggested Activity: Group Activity**

The entire class is divided into totally four groups. Each group is assigned a specific topic and asked to discuss about various points involved in that topic.

* **Group-1:**

The first group is asked to discuss about the long division method and the students involved with more interest and also the steps involved is discussed.

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* **Group-2:**

The second group is asked to discuss about the partial fraction expansion method and the students involved with more interest and also the steps involved is discussed.

* **Group-3:**

The third group is asked to discuss about the residue method and the students involved with more interest and also the steps involved is discussed.

* **Group-4:**

The fourth group is asked to discuss about the convolution method and the students involved with more interest and also the steps involved is discussed.

**Conclusion: Inverse z-transform**

**Suggested Activity: Show & Tell Activity**

* X(z) = 1 / (1-1.5z-1+0.5z-2) **-------------→ Long Division method**
* X(z) = (1+z-1) / (1-z-1+0.5z-2) **--------------→ Partial Fraction Expansion method**
* X(z) = (1+z-1+z-2) / (1-z-1) **--------------→ Residue method**

**Ref :** <http://www.d-filter.ece.uvic.ca/SupMaterials/Slides/DSP-Ch03-S8.pdf>

**Session-4 Date: 31.07.13 , 4th hour, Time: 12.00 pm-12.50 pm**

**Recap: Inverse z-transform**

**Suggested Activity: Quiz**

1. **Define inverse z-transform.**

X(z) $=\sum\_{n=-\infty }^{\infty }cn z^{-n}$

1. **List the different forms inverse z-transform.**
2. Long Division method
3. Partial Fraction Expansion method
4. Residue method
5. Convolution method
6. **Give another name for long division method.**

Power Series Expansion

**Content: Difference equation & Solution by Z-transform**

**Suggested Activity:** Board Activity

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**Conclusion: Difference equation & Solution by Z-transform**

**Suggested Activity:** Remembering



**Ref:** <http://www.wolframalpha.com/input/?i=inverse+Z+transform+calculator>

**Session-5 Date: 02.08.13, 1st hour, Time: 09.15 am-10.05 am**

**Recap: Difference equation & Solution by Z-transform**

**Suggested Activity:** Brain Storming

1. **Trial solution method : ![[Graphics:Images/ZTransformIntroMod_gr_436.gif]]()**
2. **z-transform method : ![[Graphics:Images/ZTransformIntroMod_gr_441.gif]]()**
3. **Residue method :**

**![[Graphics:Images/ZTransformIntroMod_gr_442.gif]]()**

**Content: Application to discrete systems**

**Suggested Activity:** Match the following

 Column-A Column-B

|  |  |
| --- | --- |
|  1. Real part | \operatorname{Re}\{x[n]\}\  |
|  2. Imaginary part | \operatorname{Im}\{x[n]\}\  |
|   |  |

  (4)

  (3)
 3. Convolution ![\frac{1}{2j}\left[X(z)-X^*(z^*) \right]]() (2)

 4. Parseval's relation ![\frac{1}{2}\left[X(z)+X^*(z^*) \right]]() (1)

**Conclusion: Application to discrete systems**

**Suggested Activity:** Recall by Keywords

 (i) If ![[Graphics:Images/ZTransformIntroMod_gr_205.gif]]()has a simple pole at  ![[Graphics:Images/ZTransformIntroMod_gr_206.gif]](),  then the residue is

            ![[Graphics:Images/ZTransformIntroMod_gr_207.gif]]().

(ii) If ![[Graphics:Images/ZTransformIntroMod_gr_208.gif]]()has a pole of order ![[Graphics:Images/ZTransformIntroMod_gr_209.gif]]()at  ![[Graphics:Images/ZTransformIntroMod_gr_210.gif]](),  then the residue is

            ![[Graphics:Images/ZTransformIntroMod_gr_211.gif]]().

(iii)      If ![[Graphics:Images/ZTransformIntroMod_gr_212.gif]]()has a pole of order ![[Graphics:Images/ZTransformIntroMod_gr_213.gif]]()at  ![[Graphics:Images/ZTransformIntroMod_gr_214.gif]](),  then the residue is

            ![[Graphics:Images/ZTransformIntroMod_gr_215.gif]]().

Ref: <http://lpsa.swarthmore.edu/ZXform/InvZXform/InvZXform.html>

**Session-6 Date: 03.08.13 , 6th hour, Time: 2.20 pm-3.10 pm**

**Recap: Application to discrete systems**

**Suggested Activity:** Group Discussion

The entire class is divided into totally three groups. Each group is assigned a specific topic and asked to discuss about various points involved in that topic.

* **Group-1: Common Impulse Responses**

The first group is asked to discuss about the Common Impulse Responses.

**Delta Function:**

The simplest impulse response is nothing more that a delta function, an impulse on the input produces an identical impulse on the output. This means that *all* signals are passed through the system *without change*. Convolving any signal with a delta function results in exactly the same signal Mathematically, this is written: x(n)\*δ(n) =x(n).

* **Group-2: Commutative Property**

The commutative property for convolution is expressed in mathematical form:

*a*[*n*] \* *b*[*n*] = *b*[*n*] \* *a*[*n*],

 the order in which two signals are convolved makes no difference; the results are identical. In any linear system, the input signal and the system's impulse response can be *exchanged* without changing the output signal.

* **Group-3: Distributive Property**

In equation form, the distributive property is written:

*a*[*n*]\**b*[*n*]+*a*[*n*]\**c*[*n*] = *a*[*n*] \* (*b*[*n*] + *c* [*n*] )

 The distributive property describes the operation of **parallel systems with added**

 **outputs**. The distributive property allows this combination of systems to be replaced with

 a single system, having an impulse response equal to the *sum* of the impulse responses of

 the original systems.

**Content: Stability analysis**

**Suggested Activity:** Brainstorming

1. **Stability**

 It can be shown that for any system with a transfer function H(z), all the poles of H(z)

 must lie within the unit-circle on the z-plane for the system to be stable. Zeros of the

 transfer function may lie inside or outside the circle.

1. **Gain**

Gain is the factor by which the output magnitude is different from the input magnitude. If the input magnitude is the same as the output magnitude at a given frequency, the filter is said to have "unity gain".

1. **Application to signal processing**

Digital signal processing often involves the design of finite impulse response (FIR) filters.  A simple 3-point FIR filter can be described as

   ![[Graphics:Images/ZTransformIntroMod_gr_325.gif]]().

**Conclusion: Stability analysis**

**Suggested Activity:** Question & Answers

1. Give the condition for the system to be stable.

$ \sum\_{n=-\infty }^{\infty }h\left(n\right)< \infty $

1. **What is the equation for one sided transform?**

$ X^{+}\left(z\right)≡ \sum\_{n=0}^{\infty }x\left(n\right)z^{-n}$

1. Define **First Order Difference Equations.**

The solution of difference equations is analogous to the solution of differential

 equations.

             ![[Graphics:Images/ZTransformIntroMod_gr_384.gif]]()
Ref:" <http://ocw.mit.edu/resources/res-6-008-digital-signal-processing-spring-2011/study-materials/MITRES_6_008S11_lec06.pdf>

**Session-7 Date: 07.08.13 , 1st hour, Time: 9.00 am-10.05 am**

**Recap: Stability analysis**

**Suggested Activity:** Group Activity

The entire class is divided into totally three groups. Each group is assigned a specific topic and asked to discuss about various points involved in that topic.

* **Group-1: Causality**

The first group is asked to discuss about the causality of the system, (i.e), a linear time –invariant system is causal if and only if the ROC of the system function is the exterior of a circle of radius r < ∞, including the point z = ∞.

* **Group-2: Stability**

The second group is asked to discuss about the stability of the system, the necessary and sufficient condition for a linear time –invariant system to be BIBO stable is

$$\sum\_{n=-\infty }^{\infty }h\left(n\right)< \infty $$

* **Group-3:** One-sided z-Transform

The two sided z-transform requires the signals to be in the range -∞ < n < ∞. The one-sided z-transform can be used to solve difference equations with initial conditions.

$$X^{+}\left(z\right)≡ \sum\_{n=0}^{\infty }x\left(n\right)z^{-n}$$

**Content: Frequency response**

**Suggested Activity:** Tit for Tat

**(i)First order systems**

The first order system contains only one pole and the z-transform is given by,

X(z) = 1/ (1-z-1), which contains a pole at z=1.

Therefore the output signal has the transform, Y(z) = H(z)X(z).

**(ii)Second order systems**

The second order difference equation is given by,

Y(n) = -a1 y(n-1) - a2 y(n-2) + b0 x(n)

The system function is

H(z) = Y(z)/X(z) = b0 / (1 + a1 z-1 + a2 z-1)

 = b0z2 / (z2+ a1 z +a2)

**(iii)Steady state and transient response**

The natural response is also called as the **transient response,** and it has the form

$$y\_{nr}\left(n\right)= \sum\_{k=1}^{N}A\_{k}p\_{k}^{n} u(n)$$

pk, k-1,2,…..,N are the poles of the system

Ak are scale factors of the system.

The forced response of the system is also termed as the **Steady state response,** and ithas the form,

$$y\_{fr}\left(n\right)= \sum\_{k=1}^{L}Q\_{k}q\_{k}^{n}u(n)$$

qk, k-1,2,…..,N are the poles of the forcing function

Qk are scale factors of the system.

**Conclusion: Frequency response**

**Suggested Activity:** Unspoken word

1. Natural Response: Transient response of the causal system.
2. Forced Response : Steady state response of the causal system.
3. Three different cases:
4. Real and distinct poles
5. Real and equal poles
6. Complex-conjugate poles

Ref:" <http://ocw.mit.edu/resources/res-6-008-digital-signal-processing-spring-2011/study-materials/MITRES_6_008S11_lec06.pdf>

**Session-8 Date: 07.08.13 , 3rd hour, Time: 11.10 am-12.00 pm**

**Recap: Frequency response**

**Suggested Activity:** Quiz

1. Natural Response is also called as Transient response
2. Forced Response is also called as Steady state response
3. Pole-Zero cancellations- z-transform has a pole at the same location as a zero, the pole is cancelled by a zero.

**Content: Fourier transform of discrete sequence**

**Suggested Activity:** Board Activity

The z-transform of the unit-step sequence

![[Graphics:Images/ZTransformIntroMod_gr_84.gif]]() is   ![[Graphics:Images/ZTransformIntroMod_gr_85.gif]]().

![[Graphics:Images/ZTransformIntroMod_gr_86.gif]]()

![[Graphics:Images/ZTransformIntroMod_gr_87.gif]]()

**Discrete Fourier Transform:**

The discrete time signal x(n) is converted into an equivalent frequency domain representation X(ω). Such a frequency domain representation X(ω) leads to the Discrete Fourier Transform (DFT).

**Conclusion: Fourier transform of discrete sequence**

**Suggested Activity:** Question & Answers

1. Define DFT.

It is a powerful computational tool for performing frequency analysis of discrete-time signals.

1. **Give the equation for the cos function.**

The z-transform of the sequence   ![[Graphics:Images/ZTransformIntroMod_gr_142.gif]]()   is   ![[Graphics:Images/ZTransformIntroMod_gr_143.gif]]().

**Ref:** <http://en.wikibooks.org/wiki/Digital_Signal_Processing/Sampling_and_Reconstruction>

**Session-9 Date: 16.08.13 , 2nd hour, Time: 10.05 am-10.55 am**

**Recap: Fourier transform of discrete sequence**

**Suggested Activity:** Remembering

1. Converting time domain to frequency domain is DFT.
2. It is used for analysis of discrete time signals.
3. It is an equivalent frequency domain specification.

**Content: Discrete Fourier series**

**Suggested Activity:** Board Activity

1. **Discrete Linear Convolution**

 Discrete linear convolution is the operation performed by a discrete filter, say *T*, to

 obtain an output *y*[*n*] , when an input *x*[*n*].

1. **Discrete Cyclic Convolution**

 The cyclic convolution is an operation performed on two periodic sequences, say *x*~[*n*]

 and *h*[*n*] , with the same fundamental period of length, say *N* . The result of the cyclic

 convolution is a new periodic sequence or discrete signal, say *y*~[*n*] , with the same

 fundamental period of length*N* . Thus, the cyclic convolution of *x*~[*n*] and *h*[*n*] is a new

 sequence *y*~[*n*].

1. **Time-Domain Cyclic Convolution Theorem**

 The discrete Fourier transform (*DFT*) of the cyclic convolution of two sequences, say

 *x*[*n*] and *h*[*n*] , is equal to the product of the discrete Fourier transforms of the individual

 sequences.

**Conclusion: Discrete Fourier series**

**Suggested Activity:** Jumbled words

1. Noitulovnoc cilcyc - Cyclic convolution
2. Morfsnart reiruof- Fourier transform

**Ref:** <http://www.youtube.com/watch?v=G-J4FG2L_yc>

**Ref:** <http://www.songho.ca/dsp/signal/signals.html>

**Session-10 Date: 16.08.13 , 3rd hour, Time: 11.10 am-12.00 pm**

**Content: Tutorial based on Z-transform and its properties**

**Suggested Activity:** Board Activity- Problem Solving

1. Z-transform X(z) = $\sum\_{n=-\infty }^{\infty }x(n)z^{-n}$

**Session-11 Date: 17.08.13 , 6th hour, Time: 2.20 am-3.10 pm**

**Content: Tutorial based on Inverse Z-transform**

**Suggested Activity:** Board Activity- Problem Solving

1. Long Division method
2. Partial Fraction Expansion method

**Session-12 Date: 21.08.13 , 3rd hour, Time: 11.10 am-12.00 pm**

**Content: Tutorial based on Inverse Z-transform**

**Suggested Activity:** Board Activity- Problem Solving

1. Convolution method
2. Residue method

**Session-13 Date: 21.08.13 , 4th hour, Time: 12.00 pm-12.50 pm**

**Content: Tutorial based on Inverse Z-transform**

**Suggested Activity:** Board Activity- Problem Solving

1. Solution to differential equations.

**Ref:** <http://www.wolframalpha.com/input/?i=inverse+Z+transform+calculator>

**Ref:** <http://ocw.mit.edu/resources/res-6-008-digital-signal-processing-spring-2011/study-materials/MITRES_6_008S11_lec06.pdf>