**Unit-III**

**Discrete Fourier Transform & Computation**

**Session-1 Date: 29.08.13 , 1st hour, Time: 9.15 am-10.05 am**

**Recap: Fourier transform of discrete sequence**

**Suggested Activity: Quiz**

1. A discrete-time signal of fundamental period consists of **2π / N radians.\**
2. The frequency range for continuous-time signals extends from -∞ to ∞.
3. Continuous-time periodic signals are called **power signals.**

**Content: Introduction to DFT properties**

**Suggested Activity: Board Activity**

1. **Linearity:**

If x1(n) ↔ X1(ω) and

x2(n) ↔ X2(ω)

then, a1x1(n) + a2x2(n) ↔ a1 X1(ω) + a2 X2(ω)

1. **Time Shifting:**

If x(n) ↔ X(ω)

then, x(n-k) ↔ e-jωk X(ω)

1. **Time Reversal:**

If x(n) ↔ X(ω)

then, x(-n) ↔ X(-ω)

**Ref:**<http://eeweb.poly.edu/iselesni/EL713/zoom/dftprop.pdf>

**Conclusion: Introduction to DFT properties**

**Suggested Activity:** Rapid fire

1. The signal x(t) has a finite number of finite discontinuities.
2. The signal x(t) has a finite number of maxima and minima.
3. The signal x(t) is absolutely integrable.

**Ref:** <http://en.wikipedia.org/wiki/Discrete_Fourier_transform>

**Session-2 Date: 30.08.13 , 2nd hour, Time: 10.05 am-10.55 am**

**Recap: Introduction to DFT properties**

**Suggested Activity: Show & Tell Activity**

* a1x1(n) + a2x2(n) ↔ a1 X1(ω) + a2 X2(ω) ----→ **Linearity**
* x(n-k) ↔ e-jωk X(ω) ----→ **Time Shifting**
* x(-n) ↔ X(-ω) ----→ **Time Reversal**

**Content: DFT properties**

**Suggested Activity:** Brain Storming

* Real signals:

XR(ω) = ωn

XI(ω) = ωn

* Real and even signals:

x(n) = 1/πcos

* Real and odd signals:

x(n) = -1/πsin

**Conclusion: DFT properties**

**Suggested Activity:** Pick & Answer

There is variety of questions based on the content of the session and any one of the learner is asked to pick the letter and the corresponding question to be answered**.**

1. Real signals
2. Real and even signals
3. Real and odd signals
4. Purely imaginary signals

If the learner choose **letter d** then the question isPurely imaginary signals and the answer is

x(n) = 1/πcos

**Ref :** <http://licos.epfl.ch/courses/dsp0607/dsp_n02_chapter2.pdf>

**Session-3 Date: 30.08.13 , 4th hour, Time: 12.00 pm-12.50 pm**

**Recap: DFT properties**

**Suggested Activity: Quiz**

1. **Convolution theorem:**

x(n)= x1(n)\*x2(n)↔X(ω) = X1(ω)X2(ω)

1. **Correlation theorem:**

rx1x2(m) ↔ Sx1x2(ω) = X1(ω)X2(-ω)

**Content: Magnitude representation**

**Suggested Activity:** Group Activity

The entire class is divided into totally three groups. Each group is assigned a specific topic and asked to discuss about various points involved in that topic.

* **Group-1:**

The first group is asked to discuss about the Circular shift of a sequence and the students involved with more interest and also the steps involved is discussed.

.

* **Group-2:**

The second group is asked to discuss about the Circular convolution and the students involved with more interest and also the steps involved is discussed.

* **Group-3:**

The third group is asked to discuss about the Circular correlation and the students involved with more interest and also the steps involved is discussed.

**Conclusion: Magnitude representation**

**Suggested Activity:** Unspoken word

1. **Equalizer:** In digital communications, a corrective system.
2. **System identification:** A set of measurements performed on the system.
3. **Invertible:** One-to-one correspondence between its input and output signals.

**Ref :** <http://cacr.uwaterloo.ca/techreports/2012/cacr2012-21.pdf>

**Session-4 Date: 30.08.13 , 5th hour, Time: 1.30 pm-2.20 pm**

**Recap: Magnitude representation**

**Suggested Activity: Board Activity**

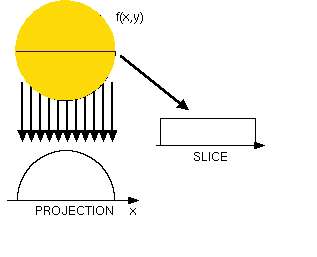
**Content: Phase representation**

**Suggested Activity:** Group Activity

The entire class is divided into totally two groups. Each group is assigned a specific topic and asked to discuss about various points involved in that topic.

* **Group-1:**

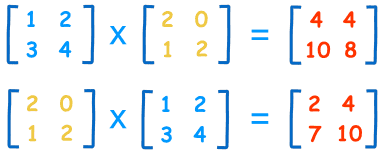
The first group is asked to discuss about the concentric circle method and the students involved with more interest and also the steps involved is discussed.

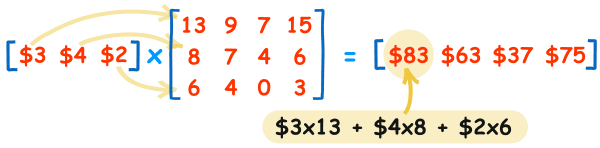
.

* **Group-2:**

The second group is asked to discuss about the matrix multiplication method and the students involved with more interest and also the steps involved is discussed.

http://www.mathsisfun.com/algebra/images/matrix-multiply-rows-cols.gif



****

**Ref:**<http://en.wikipedia.org/wiki/Matrix_multiplication>

**Conclusion: Phase representation**

**Suggested Activity:** Questions & Answers

1. Convolution concept

Y(n)

1. Impulse Fourier transform

H(F) = e-j2πFt dt

**Ref:** <http://www.sciencedirect.com/science/article/pii/0010448592900359>

**Session-5 Date: 31.08.13, 6th hour, Time: 2.20 pm-3.10 pm**

**Recap: Phase representation**

**Suggested Activity:** Brain Storming

1. HR(ω) = ------→ Real component of H(ω)
2. HI(ω) = ------→ Imaginary component of H(ω)
3. │H(ω)│=
4. Angle of H(ω) = tan-1[HI(ω) / HR(ω)]

**Content: Computation of DFT using FFT algorithm**

**Suggested Activity: Board activity**

Two different approaches are used to find the DFT of the sequence. The important computational algorithm called Fast Fourier Transform (FFT) algorithms, for computing the DFT when the size N is a power of 2 and when it is a power of 4.

Computation of DFT using FFT Algorithms

FFT : Fast Fourier Transform Algorithms

1. X(k) = WNnk
2. Direct Computation of DFT
3. To compute one DFT point
4. N complex multiplications are required
5. N-1 complex additions are required
6. To compute N DFT points (k = 0,1, …,N-1)
7. N2 complex multiplications (N x N)
8. N2-N complex additions. ( (N -1) x N )

**Conclusion: Computation of DFT using FFT algorithm**

**Suggested Activity:** Recall by Keywords

1. **DFT :** Discrete Fourier Transform
2. FFT : Fast Fourier Transform
3. **Separability:**

http://bme.med.upatras.gr/improc/mod1-18.gif

http://bme.med.upatras.gr/improc/mod1-19.gif

1. **Periodicity and Conjugate Symmetry**

http://bme.med.upatras.gr/improc/proper9.jpg

Ref: <http://www.viit.ac.in/FFT_Algorithm.pdf>

**Session-6 Date: 02.09.13 , 6th hour, Time: 2.20 pm-3.10 pm**

**Recap: Computation of DFT using FFT algorithm**

**Suggested Activity: Remembering**

1. **Rotation:** Simply stated: if a function is rotated, then its Fourier transform rotates an equal amount.



1. **Distributivity :** The Fourier transform and its inverse are distributive over addition but not over multiplication.

http://bme.med.upatras.gr/improc/proper10.jpg

**http://bme.med.upatras.gr/improc/proper11.jpg**

**Content: Decimation-in-time algorithm**

**Suggested Activity:** Group Discussion

A decimation-in-time radix-2 FFT breaks a length-*N* DFT into two length-*N*/2 DFTs followed by a combining stage consisting of many butterfly operations. A decimation-in-time FFT algorithm on *n* = 2*p* inputs with respect to a primitive *n*-th root of unity *ω* = exp(2*πi* / *n*) relies on O(*n* log *n*) butterflies of the form:

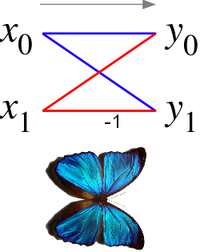
y_0 = x_0 + x_1 \omega^k \, 

y_1 = x_0 - x_1 \omega^k, \, 

**Conclusion: Decimation-in-time algorithm**

**Suggested Activity:** Show & Tell Activity

**Radix-2 butterfly diagram**

[](http://en.wikipedia.org/wiki/File:Butterfly-FFT.png)

Inversion of butterflies:

x_0 = \frac{1}{2} (y_0 + y_1) \, 

x_1 = \frac{\omega^{-k}}{2} (y_0 - y_1), \, 

Ref: <http://en.wikipedia.org/wiki/Butterfly_diagram>

**Session-7 Date: 03.09.13 , 6th hour, Time: 2.20 pm-3.10 pm**

**Recap: Decimation-in-time algorithm**

**Suggested Activity:** Quiz

1. Symmetry Property: WNk+N/2 = -WNk
2. Periodicity Property: WNk+N = WNk
3. Direct computation requires
4. 2N2 evaluations of trigonometric functions
5. 4N2 real multiplications
6. 4N(N-1) real additions
7. A number of indexing and addressing operations

**Content: Decimation-in-frequency algorithm**

**Suggested Activity:** Group Discussion

The entire class is divided into totally two groups. Each group is assigned a specific topic and asked to discuss about various points involved in that topic.

* **Group-1:**

**equation**

* **Group-2:**

**equation**

* **Group-3:**

equation

**Conclusion: Decimation-in-frequency algorithm**

**Suggested Activity:** Questions & Answers

1. Define DIF algorithm.

The process of dividing the frequency components into even and odd parts is what gives this algorithm its name 'Decimation In Frequency'.

1. **What is meant by 'twiddle factors'?**

The factors *TN* are conventionally referred to as 'twiddle factors'.

Ref: <http://www.systems.caltech.edu/EE/Courses/EE32b/handouts/FFT.pdf>

**Session-8 Date: 04.09.13 , 3rd hour, Time: 11.10 am-12.00 pm**

**Recap: Decimation-in-frequency algorithm**

**Suggested Activity:** Brainstorming

1. **Twiddle factors:** equation
2. **Pass Loop:** An *N* (=2*p* ) point transform will perform *p* 'passes', indexed by *P*=0..*p*-1.
3. **Block Loop:** Pass will *P* operate on *BP* (=2*P* ) sub-blocks, each of size *NP* (=*N*/*BP*=2*p*-*P*), indexed by b=0..*BP*-1
4. **Butterfly Loop:** Each sub-block operation will perform *N*'*P* (=*NP*/2=2*p*-*P-1*) butterflies, indexed by n=0..*N*'*P*-1.

**Content: FFT using radix 2**

**Suggested Activity:** Group Activity

equation equation

equation

equation

equation

**Conclusion: FFT using radix 2**

**Suggested Activity:** Pick & Answer

There are variety of questions based on the content of the session and any one of the learner is asked to pick the letter and the corresponding question to be answered**.**

1. **Radix-2 FFT algorithms**
2. **Radix-4 FFT algorithms**
3. **Symmetry property**
4. **Periodicity property**

If the learner choose **letter d** then the question is **Periodicity property** and the answer is WNk+N = WNk.

**Ref:** <http://www.engineeringproductivitytools.com/stuff/T0001/PT03.HTM>

**Session-9 Date: 04.09.13 , 4th hour, Time: 12.00 pm-12.50 pm**

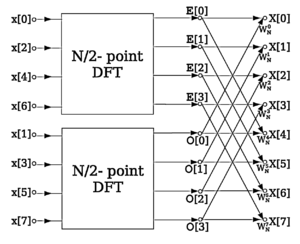
**Recap: FFT using radix 2**

**Suggested Activity:** Quiz

1. XR(k) = xR(n) + sin()xI(n)]
2. XI(k) = xR(n) - xI(n)]

**Content: Construction of 8 point DFT**

**Suggested Activity:** Board Activity

[](http://en.wikipedia.org/wiki/File:DIT-FFT-butterfly.png)

The butterfly can also be used to improve the randomness of large arrays of partially random numbers, by bringing every 32 or 64 bit word into causal contact with every other word through a desired hashing algorithm, so that a change in any one bit has the possibility of changing all the bits in the large array.

**Conclusion: Construction of 8 point DFT**

**Suggested Activity:** Questions & Answers

1. 1st Algorithm for computation of DFT:
2. Store the signal column-wise
3. Compute the M-point DFT of each row
4. Multiply the resulting array by the phase factors WNlq
5. Compute the L-point DFT of each column.
6. Read the resulting array row-wise.
7. 2nd Algorithm for computation of DFT:
8. Store the signal column-wise
9. Compute the L-point DFT at each column
10. Multiply the resulting array by the phase factors WNpm
11. Compute the M-point DFT of each row.
12. Read the resulting array column-wise.

**Ref:** <http://sist.sysu.edu.cn/uploaded/file/Chpt04%281%29.pdf>

**Session-10 Date: 05.09.13 , 1st hour, Time: 9.15 am-10.05 am**

**Content: Tutorial: Computation of DFT using FFT algorithm**

**Suggested Activity:** Board Activity- Problem Solving

**Ref:** <http://www.sciencedirect.com/science/article/pii/0010448592900359>

**Session-11 Date: 06.09.13 , 2nd hour, Time: 10.05 am-10.55 am**

**Content: Tutorial: Decimation-in-time algorithm (DIT)**

**Suggested Activity:** Board Activity- Problem Solving

**Ref:** <http://www.mathworks.in/help/matlab/math/fast-fourier-transform-fft.html>

**Session-12 Date: 13.09.13 , 2nd hour, Time: 10.05 am-10.55 am**

**Content: Tutorial: Decimation-in-time algorithm (DIF)**

**Suggested Activity:** Board Activity- Problem Solving

**Ref:** <http://www.numerix-dsp.com/tutorials/DSP/FrequencyDomainProcessing.pdf>